ELSEVIER

SCIENCE DIRECT®

PHYSICS LETTERS B

Physics Letters B 564 (2003) 1–7

www.elsevier.com/locate/npe

Unusual features of varying speed of light cosmologies

John D. Barrow

DAMTP, Centre for Mathematical Sciences, Cambridge University, Cambridge CB3 0WA, UK

Received 22 November 2002; received in revised form 8 April 2003; accepted 9 April 2003

Editor: P.V. Landshoff

Abstract

We contrast features of simple varying speed of light (VSL) cosmologies with inflationary universe models. We present new features of VSL cosmologies and show that they face problems explaining the cosmological isotropy problem. We also find that if c falls fast enough to solve the flatness and horizon problems then the quantum wavelengths of massive particle states and the radii of primordial black holes can grow to exceed the scale of the particle horizon. This may provide VSL cosmologies with a self-reproduction property. The constraint of entropy increase is also discussed. The new problems described in the this Letter provide a set of bench tests for more sophisticated VSL theories to pass.

1. Introduction

We have explored a naive class of models in which the speed of light varies in time [1,2]. These can be reformulated more generally and recast as theories in which other dimensional 'constants' carry the operationally meaningful space or time variation of a dimensionless constant—in this case the fine structure 'constant', $e^2/\hbar c$, [3]. Theories in which this variation is carried explicitly by the electron change have been extensively investigated recently [4–10]. Such theories are a 'small' perturbation of standard physics in the sense that even though the speed of light falls there still exists a maximum signal propagation velocity that is achieved by gravitational waves. The motivation for a careful consideration of these cosmological models was their possible viability as alternatives to inflation as explanations for a number of unusual properties of the universe and the consistency of quasar absorption spectra with a variation in the value of the fine structure 'constant' at $z \sim 1$–3, [13–16]. In particular, it was shown by Moffat [11,12], Albrecht and Magueijo [1] and by Barrow [2] that during the very early universe a finite period of time during which the speed of light falls at an appropriate rate can lead to a solution of the flatness, horizon, and monopole problems. However, unlike inflation it can also provide a solution for the cosmological constant problem. Barrow and Magueijo also showed that for some ranges of variation in the speed of light these theories can also naturally create long-lived universe in which the dynamics are almost flat or in which the cosmological constant is almost zero. These problems of explaining a universe in which the present value of the matter density parameter, Ω_m, or the cosmological constant energy density parameter, Ω_Λ, are $O(0.1$–$1)$ we called the quasi-flatness and quasi-lambda problems. Inflation does not seem to offer a natural explanation of a universe which is almost flat or has a dynamically

E-mail address: j.d.barrow@dampt.cam.ac.uk (J.D. Barrow).

0370-2693 © 2003 Elsevier Science B.V. Open access under CC BY license.
doi:10.1016/S0370-2693(03)00573-2

significant cosmological constant today, as current observations imply.

In this Letter, we want to discuss some other cosmological problems in the light of varying speed of light (VSL) theories of the simple sort discussed by Albrecht, Magueijo, and Barrow (AMB). Although these problems are articulated in the context of this simple AMB theory we believe that are more general challenges to any theory which manifests itself in a particular coordinate system as a VSL theory. In particular, we shall show that in general AMB theories cannot solve the isotropy problem and are unable to generate a spectrum of almost constant curvature fluctuations in the standard way. We shall also show that they have a number of very unusual consequences for the quantum states of massive particles and for primordial black holes after they enter the particle horizon in the early universe. It is not clear whether these features are a reductio ad absurdum for these VSL theories or whether they provide an exotic counterpart to the self-reproducing property of inflationary universe models that provides the basis for eternal inflationary universes.

2. Naive VSL theories

2.1. Solving the flatness, horizon and lambda problems

The simple VSL model introduced by Albrecht and Magueijo and solved by Barrow is based upon the simplest possible premises. We assume that the Friedmann cosmology in the presence of time-dependent speed of light $c(t)$ is described by the Friedmann and Raychaudhuri equations (although the original formulation allowed the Newtonian gravitation 'constant' G to vary also, we shall ignore this possibility as it makes no contribution to the essential conclusions):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2(t)}\right) + \frac{\Lambda c^2(t)}{3}, \tag{1}$$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{kc^2(t)}{a^2} + \frac{\Lambda c^2(t)}{3}, \tag{2}$$

where $a(t)$ is the expansion scale factor of the Friedmann metric, p is the fluid pressure, ρ is the fluid density, k is the curvature parameter, Λ is the cosmological constant, and all derivatives are with respect to t, the comoving proper time. From these equations we can derive the modified matter conservation which incorporates the effects of $\dot{c} \neq 0$:

$$\dot{\rho} + \frac{3\dot{a}}{a}\left(\rho + \frac{p}{c^2(t)}\right) = \frac{3kc^2\dot{c}}{4\pi Ga^2}. \tag{3}$$

Of course, in general relativity, where $\dot{c} = 0$, the right-hand side of (3) is zero. Notice that it also vanishes in a VSL theory when $k = 0$. For concreteness, now consider the universe to contain a perfect fluid with equation of state

$$p = (\gamma - 1)\rho c^2, \tag{4}$$

where γ is constant and we shall assume that the speed of light varies as some power of the expansion scale factor:

$$c(t) = c_0 a^n, \tag{5}$$

where $c_0 > 0$ and n are constants. These assumptions are sufficient to solve the equations completely. Integrating (3) with $\Lambda = 0$, we find

$$\rho = \frac{M}{a^{3\gamma}} + \frac{3kc_0^2 n a^{2n-2}}{4\pi G(2n - 2 + 3\gamma)}. \tag{6}$$

Now by inspection of (2) we see that at large a the curvature term falls off faster than the $Ma^{-3\gamma}$ term whenever

$$n < \frac{1}{2}(2 - 3\gamma). \tag{7}$$

Thus the flatness problem can be solved in a radiation-dominated early universe by an interval of VSL evolution if

$$n < -1.$$

It is easy to show that, as in the case of inflation, this is the same condition required to solve the monopole and horizon problems. However, the nature of the solution differs from that provided by a period of inflation. Inflation requires the dynamics to be dominated for a finite period by an unusual fluid ($\gamma < -2/3$), which is therefore gravitationally repulsive, so that the expansion will accelerate ($\ddot{a} > 0$) and the $Ma^{-3\gamma}$ term will grow to dominate the kc^2a^{-2} term in (2) as $a(t)$ grows large. By contrast, in the VSL model, no special fluid is required and in a radiation universe the flatness problem is solved because the kc^2a^{-2} term falls off faster than the $Ma^{-3\gamma}$ term because of the fall in $c(t)$ as $a(t)$ increases.

This behaviour also permits a solution of the classical cosmological constant problem in just the same way if

$$n < -\frac{3\gamma}{2}.$$

When $\Lambda > 0$ we can solve the system of equations to obtain the density:

$$\rho = \frac{M}{a^{3\gamma}} + \frac{3kc_0^2 na^{2n-2}}{4\pi G(2n-2+3\gamma)} - \frac{\Lambda c_0^2 na^{2n}}{4\pi G(2n+3\gamma)}.$$

In this case the Λc^2 term falls off faster with increasing time than the $Ma^{-3\gamma}$ term and the dynamics naturally approach those of the $k = 0 = \Lambda$ Friedmann model at late times. A period of radiation-dominated evolution with

$$n < -2,$$

can therefore solve an initial classical lambda problem.

Inflationary universe models do not offer a solution of this problem. It is interesting that the flatness and cosmological constant problems can be solved by these VSL theories within any fine tuning of n. But note also that even in the VSL models there is no solution of the non-classical Λ problem because a sequence of phase transitions are able to reinstate the value of the cosmological constant at successive cosmological epochs even if its value is set equal to zero at some arbitrarily early time.

The simple mathematical model for VSL considered here is oversimplified in many respects but it can easily be put on a former foundation. If we define a scalar field $\psi = \log(c/c_0)$ then a simple action to describe its coupling to gravity is the choice

$$S = \int d^4x\,\sqrt{-g}\left(e^{a\psi}\left(R - \kappa(\psi)\nabla_\mu\psi\nabla^\mu\psi\right) + \frac{16\pi G}{c_0^4}e^{b\psi}L_m\right)$$

parametrized by the constants $a, b,$ and κ. Similar properties of the resulting cosmological solutions are found to those displayed by the simple theory above. Many of the potential problems raised in the Letter need to be readdressed in the context of a specific theory, defined by a given Lagrangian [17–24]. At present there is no unique VSL theory. However, the naive VSL theory described above plays an important role in generating hypotheses to be tested in more sophisticated theories.

2.2. The quasi-flatness and quasi-lambda problems

However, although string theories seem to predict that the cosmological constant should be zero, there is solid observational evidence that it is small and positive and dominating the expansion dynamics of the universe today ($\Omega_m \sim 0.3, \Omega_\Lambda \sim 0.7$). These observations lead us to ask whether there can be ‘natural’ explanations for universes which are ‘almost’ flat or have ‘almost’ zero cosmological constant. Barrow and Magueijo [9] have shown that these simple VSL models can provide solutions to both problems if the range of n is narrowed. Thus, when the speed of light falls as (5) with

$$0 > n > \frac{1}{2}(2 - 3\gamma) \tag{8}$$

we find that at late times we have a quasi-flat universe and

$$\Omega \to \frac{-2n}{3\gamma - 2}.$$

Similarly, when

$$0 > n > -\frac{3\gamma}{2}$$

we have a significant cosmological constant contribution to the expansion at late times with

$$\frac{\Omega_m}{\Omega_\Lambda} \to -\frac{3\gamma\Omega_m}{2(\Omega_\Lambda + \Omega_m)}.$$

3. Other consequences of VSL theories

3.1. The isotropy problem

The isotropy problem is solved by inflationary universe models with $0 \leqslant \gamma < 2/3$ because the dominant anisotropy modes in expanding anisotropic universes fall off no slower than $\sigma^2 \propto a^{-2}$ and Eq. (2) is modified in the presence of anisotropies by the addition of a σ^2 term to its right-hand side. Thus we see that at late times in an inflationary expansion the $Ma^{-3\gamma}$ term falls off more slowly than $\sigma^2 \propto a^{-2}$ and dominates the dynamics, driving the expansion away from isotropy. Note however that in generic ever-expanding universes the source of this dominant $\sigma^2 \propto a^{-2}$ or t^{-2} late-time anisotropy is the anisotropy in the 3-curvature of space. The simpler, much-studied anisotropy characteristic of Bianchi I or Kasner universes, arises from simple expansion rate anisotropy with isotropic spatial 3-curvature is sub-dominant at late times, with $\sigma^2 \propto a^{-6}$. This behaviour is of measure zero in the space of all anisotropic cosmological models. A solution of the anisotropy problem must explain why the $\sigma^2 \propto a^{-2}$ anisotropy mode has not come to dominate the expansion. This is possible for inflationary expansion dominated by a $\rho + 3p < 0$ effective stress since this falls off more slowly than σ^2 as $t \to \infty$ and the influence of the anisotropy can be made arbitrarily small for a sufficiently long period of accelerated expansion.

In the situation, with constant c, where inflation does not occur and isotropisation is not generic. The anisotropising mode is well understood in the context of homogeneous and anisotropic cosmologies. It is a Bianchi type VII_h plane gravitational wave described by an exact solution found by Lukash [25]. It is interesting to note that they can be excluded if an open universe has a finite topology [29]. In the case with the natural R^3 topology one can show that a particular family of known exact vacuum solutions of type VII_h are stable as $t \to \infty$ [26–28]. In particular, the isotropic open Friedmann universe is stable (but not asymptotically stable) with respect to these anisotropic curvature modes: that is, as $t \to \infty$, the ratio of the shear σ to mean Hubble expansion rate, $H = \dot{a}/a$, approaches a non-zero constant value. Inflation can make this constant arbitrarily close to zero.

In the simplest anisotropic universes with comoving perfect-fluid matter and isotropic 3-curvature and VSL we can repeat the approach taken in the case of isotropic universes, using the Raychaudhuri acceleration equation, its first integral, and the shear propagation equation—with $c(t)$ assumed—to derive a generalised matter conservation equation. Thus

$$\frac{d}{dt}\left(\frac{3H}{c}\right) = -3\frac{H^2}{c^2} - 2\frac{\sigma^2}{c^2} - \frac{4\pi G}{c^4}(\rho + 3p) + \Lambda,$$

$$^3R = \frac{16\pi G\rho}{c^4} - \frac{6H^2}{c^2} + \frac{2\sigma^2}{c^2} + 2\Lambda,$$

$$\frac{d}{dt}\left(\frac{\sigma}{c}\right) + 3H\frac{\sigma}{c} = 0. \tag{9}$$

Hence we see that $\sigma = \Sigma c a^{-3}$ with Σ constant, and if we were to write $^3R = 2ka^{-2}$ then

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) + \frac{c\dot{c}}{4\pi G}\left(\frac{2\Sigma^2}{a^6} - \frac{3k}{a^2}\right) = 0.$$

Using Eqs. (4) and (5) we can solve for ρ and we find that the flatness problem is solved as usual when (7) holds. We can see immediately that the shear effects are negligible with respect to the curvature and

$$\rho = \frac{\Gamma}{a^{3\gamma}} - \frac{2\Sigma^2 nc_0 a^{2n+3\gamma-6}}{(2n + 3\gamma - 6)} + \frac{6knc_0^2 a^{2n+3\gamma-2}}{(2n + 3\gamma - 2)}.$$

Hence, the shear term is always insignificant and the condition for solution of the flatness problem is the same as in an isotropic universe, (8).

In the general case we have to consider the contribution made the anisotropic 3-curvature terms and anisotropic fluid pressure and not just an anisotropic Hubble flow ($\sigma \neq 0$), as considered above. For simplicity, we neglect the anisotropic pressure effects but consider the role of anisotropic 3-curvature. The shear propagation equation becomes [30,31]:

$$\frac{h^\mu_\rho h^\nu_\sigma}{c}\frac{d}{dt}\left(\frac{\sigma^{\rho\sigma}}{c}\right) = -\frac{2H\sigma^{\mu\nu}}{c^2} - \frac{\sigma^\mu_\rho \sigma^{\nu\rho}}{c^2} - E^{\mu\nu} + \frac{2\sigma^2 h^{\mu\nu}}{3c^2},$$

$$E_{\mu\nu} = S_{\mu\nu} + \frac{H\sigma_{\mu\nu}}{c^2} - \frac{\sigma_{\mu\rho}\sigma^\rho_\nu}{c^2} + \frac{2\sigma^2 h_{\mu\nu}}{3c^2},$$

where $S_{\mu\nu}$ is the anisotropic part of the spatial 3-curvature and $h_{\mu\nu}$ is the projection tensor. Generic evolution at late times for ever-expanding anisotropic universes close to isotropy with $\Lambda = 0$ will have the scaling form $S_{\mu\nu} \sim E_{\mu\nu} \sim t^{-2}$, $Hc^{-1} \sim t^{-1}$, $H\sigma c^{-2} \sim t^{-2}$, $\sigma c^{-1} \sim t^{-1}$. Hence we $\sigma \propto H$ as in the case with constant c. However, we see that $\sigma^2 c^{-2} \sim t^{-2}$ in the generalised Friedmann equation (9) whereas the isotropic matter terms go as $\rho c^{-4} \propto a^{-3\gamma-4n}$. Thus the shear terms will dominate at late times close the Friedmann expansion, $a \propto t^{2/3\gamma}$ whenever $n < 0$ and a falling speed of light does not solve the anisotropy problem. There are several ways in which the VSL effects on anisotropy can be understood more physically. The naive VSL theories preserve the metric structure of spacetime so that gravitational-wave propagation still occurs at a maximum possible propagation speed. But light propagates with a variable speed that is less than or equal to the gravitational-wave propagation speed. Thus one can see that anisotropies that are carried by long-wavelength gravitational waves can avoid being made innocuous by a fall in the speed of light. The generic anisotropies at late time in ever-expanding Bianchi types that contain isotropic Friedmann universes are of this type.

3.2. The inhomogeneity problem

The spectrum of primordial fluctuations is the most interesting prediction that any VSL model can make because the observational evidence from the microwave background temperature fluctuations is potentially the most decisive of observational tests. Inflationary theories have been able to provide a natural explanation for our observations of an almost constant curvature spectrum of inhomogeneous fluctuations in the universe. Other competing predictions have been made by colliding brane models [32].

In inflationary universes the approximately constant curvature spectrum of inhomogeneities arises from the near de Sitter behaviour of the expansion dynamics during an inflationary epoch where $\gamma \approx 0$. In a finite time interval of c variation in a VSL theory that solves the flatness or lambda problems the expansion dynamics will be dominated by the usual $Ma^{-3\gamma}$ term. If the expansion is radiation dominated then no special inhomogeneity spectrum of the constant curvature form will be created by the VSL evolution. One way of imprinting a characteristic spectrum could be via the sudden phase transition model of VSL favoured by Moffat and Albrecht and Magueijo. Here, it would need to be shown that a constant curvature spectrum results. Again, this is a major challenge because the phase transition models of inflation achieve a constant curvature spectrum of fluctuations by virtue of their proximity to a de Sitter state in the vacuum state where the scalar field stops rolling. Recently, one attempt along these lines for the creation of a constant curvature spectrum has been made by Moffat [33], and Magueijo and Pogosian [34] have investigated the possibility that thermal fluctuations might give rise to an interesting primordial inhomogeneity spectrum as a result of modified dispersion relations or cosmological bounce at high energy.

3.3. The massive particle problem

VSL theories have two further strange properties that appear to have dramatic consequences. Suppose that the universe starts to experience a period of VSL evolution of the form (5). Let us consider the fate of a particle state of mass M that exists on a scale smaller than the particle horizon $R_{\mathrm{hor}} \sim ct$. Now if it has a de Broglie wavelength

$$\lambda = \frac{\hbar}{Mc}$$

then if c falls as $c \propto a^n \propto t^{n/2}$ in a radiation-dominated universe we have for the ratio of λ to the proper size of the particle horizon

$$R_{\mathrm{hor}} = a(t) \int \frac{c\,dt}{a(t)}$$

is given by

$$\begin{aligned} \frac{\lambda}{R_{\mathrm{hor}}} &\approx \frac{\hbar}{Mc} \frac{(n+1)}{2c_0 t^{(n+2)/2}} \\ &= \frac{(n+1)\hbar}{2Mc_0^2 t^{(2n+2)/2}} \propto t^{-n-1} \to \infty \end{aligned} \tag{10}$$

as $t \to \infty$ if $n < -1$. So if we take $n < -1$ as required to solve the flatness problem, massive particles evolve to become acausal separate universes! Note that, in particular, theories $\hbar$ may vary as well. For example, in the naive VSL theory of Refs. [1,2] $\hbar \propto c \propto a^n$ and so $\lambda/R_{\mathrm{hor}} \propto t^{-n/2-1}$ and a massive particle problem can arise when $n < 2$, as required to solve the classical cosmological constant problem.

How should be interpret this. Is is a reductio ad absurdum of the VSL or should we take a lesson from new inflation and regard our observable universe as the interior of the single particle state. This might provide a natural explanation for some of its coordinated properties and provide a reason for a special irregularity spectrum to be formed. Note that this behaviour of massive particle states ceases when the VSL evolution ends and will not be going on in the universe today. Since the early universe may contain a population of massive states inside the horizon scale when VSL evolution begins, the ensuing evolution is not dissimilar to the self-reproducing inflationary universe. There small regions create eternal inflation due to the quantum evolution dominating the classical slow roll under fairly general conditions. The rapid expansion of many VSL regions would produce collisions that might lad to unacceptable levels of inhomogeneity when they subsequently re-enter the horizon if the amount of expansion was small. But if it was large then we could find ourselves inhabiting a single VSL pre-expanded domain. This scenario may repay further detailed analysis. There are many complexities that have been ignored here, in particular relating to the rapid growth of dimensionless couplings like e^2/hc, g^2/hc, Gm^2hc as c falls, rendering all interactions strong. It may be that the avoidance of this problem is a constraint that needs to be placed on sophisticated VSL theories or it could be exploited as a new mechanism for making small local regions become large.

3.4. The primordial black hole problem

A similar fate awaits any small primordial black hole of mass M that forms on sub-horizon scales. Let us ignore Hawking evaporation and consider the ratio of the Schwarzschild radius of the black hole, R_{bh} to the horizon scale:

$$\frac{R_{\mathrm{bh}}}{R_{\mathrm{hor}}} = \frac{2GM}{c^2}\frac{(n+1)}{2c_0 t^{(n+2)/2}} \propto t^{-1-\frac{3n}{2}}.$$

So if $n < -2/3$ the black hole horizon grows faster than the particle horizon and the black hole becomes an acausal separate universe. Much of the discussion regarding the expansion of massive particle states applies to this situation also. Here there is an added interpretational uncertainty in that it is not clear what happens to the black hole when c changes. Changing c may be sufficient to stop a black hole forming. But if a Schwarzschild black hole formed when c did not change then it might be that it had to remain constant on the horizon even while c changed in the background.

3.5. An entropy problem

If we believe that we can apply the Bekenstein–Hawking entropy formula to the particle horizon of an expanding universe then the entropy inside the VSL horizon is

$$S \propto \frac{R_{\mathrm{hor}}^2}{R_{\mathrm{pl}}^2}$$

and this entropy evolves as $t^{\frac{5n+4}{2}}$ in VSL theories and hence increases only if $n > -4/5$ in the radiation era when $\hbar$ is constant. If $\hbar \propto c \propto a^n$, so $R_{\mathrm{pl}} \propto a^{-n}$, then the entropy increases only if $n > -1$. In general we might consider a suite of theories in which $\hbar c \propto a^{\lambda}$ so $R_{\mathrm{pl}} \propto a^{(\lambda-4n)/2}$ so $S \propto a^{6n-\lambda}t^2$ and entropy increases during the radiation era when $3n + 2 - \lambda/2 > 0$. Thus the solution of the flatness an lambda problems can require this entropy measure to decrease with increasing time. The Bekenstein–Hawking entropy technically applies only to event horizons but many analogous measures of 'gravitational entropy' have been proposed and it is possible that the argument framed here will have application to any better-motivated masure of gravitational entropy. This worry also besets arguments like those of Davies et al. [35] which use black hole thermodynamics to assess whether the variation of certain constants are in accord with the second law of thermodynamics. The difficulty is that the required black holes and their thermodynamic will only exist as particular solutions of a theory with varying constants—particular solutions in which those varying constants take constant values—and so the argument cannot be carried through. A specific case arises in Brans–Dicke theory. The Schwarzschild solution is a particular solution of Brans–Dicke theory and its Bekenstein–Hawking entropy is $S \propto GM^2$. One might be tempted therefore to think that any solution of Brans–Dicke gravity in which G decreases with time would therefore violate a second law of black hole thermodynamics. However, the Schwarzschild black hole is only a solution of Brans–Dicke theory when G is constant. If G is allowed to vary (as in the thought experiment designed to violate the second law) the static spherically symmetric solution of the Brans–Dicke equations is not a black hole.

4. Discussion

We have considered further properties of naive VSL cosmologies, in addition to those related to the flatness and horizon problems. There are problems explaining the isotropy of the universe in general and unusual consequences of varying c on particle de Broglie wavelengths and black holes. It is not clear whether the massive particle and black hole problems are fatal to the conception of VSL theories. At first sight they appear to create an unsatisfactory state of affairs. However, it may be that they are in effect counterparts of the self-reproduction property of inflationary universes that gives rise to the quasi-stationary eternal inflationary universe scenario. The rapid enlargement of single particle states to super-horizon scales simply provides a way of producing large scale regions that possess the coherent properties possessed by single particle states when they were on sub-horizon scales and to generate fluctuations on super-horizon scales. However, although states grow faster than the horizon they do not necessarily grow fast enough to create regions which could encompass our entire visible universe. Whether or not these features provide a reductio ad absurdum for these cosmologies remains to be explored in greater detail. Any more completely specified VSL theory will need to be judged both by its ability to solve the standard problems that inflation can resolve and by its ability to circumvent the difficulties described in this Letter.

Acknowledgements

I thank R. Maartens, J. Magueijo and J. Moffat for discussions.

References

[1] A. Albrecht, J. Magueijo, Phys. Rev. D 59 (1999) 043516.
[2] J.D. Barrow, Phys. Rev. D 59 (1999) 043515.
[3] J.D. Barrow, J. Magueijo, Phys. Lett. B 443 (1998) 104.
[4] H.B. Sandvik, J.D. Barrow, J. Magueijo, Phys. Rev. Lett. 88 (2002) 031302.
[5] J.D. Barrow, H.B. Sandvik, J. Magueijo, Phys. Rev. D 65 (2002) 063504.
[6] J.D. Barrow, H.B. Sandvik, J. Magueijo, Phys. Rev. D 65 (2002) 123501.
[7] J.D. Barrow, J. Magueijo, H.B. Sandvik, astro-ph/0202129.
[8] J. Magueijo, J.D. Barrow, H.B. Sandvik, astro-ph/0202374, Phys. Lett. B, in press.
[9] J.D. Barrow, D. Mota, gr-qc/0207012, Class. Quantum Grav. 19, in press.
[10] J.D. Barrow, J. Magueijo, Phys. Lett. B 447 (1999) 246.
[11] J.W. Moffat, Int. J. Mod. Phys. D 2 (1993) 351.
[12] J.W. Moffat, astro-ph/0109350.
[13] M. Murphy, J. Webb, V. Flambaum, V. Dzuba, C. Churchill, J. Prochaska, A. Wolfe, Mon. Not. R. Astron. Soc. 327 (2001) 1208.
[14] J.K. Webb, M.T. Murphy, V.V. Flambaum, V.A. Dzuba, J.D. Barrow, C.W. Churchill, J.X. Prochaska, A.M. Wolfe, Phys. Rev. Lett. 87 (2001) 091301.
[15] J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, J.D. Barrow, Phys. Rev. Lett. 82 (1999) 884.
[16] M.T. Murphy, J.K. Webb, V.V. Flambaum, S.J. Curran, astro-ph/0210532.
[17] G. Dvali, M. Zaldarriaga, hep-ph/0108217.
[18] T. Banks, M. Dine, M.R. Douglas, hep-ph/0112059.
[19] P. Langacker, G. Segre, M. Strassler, hep-ph/0112233;
X. Calmet, H. Fritzsch, hep-ph/0112110.
[20] K. Olive, M. Pospelov, hep-ph/0110377.
[21] J.D. Bekenstein, Phys. Rev. D 25 (1982) 1527.
[22] J. Magueijo, Phys. Rev. D 62 (2000) 103521.
[23] J. Magueijo, Phys. Rev. D 63 (2001) 043502.
[24] C. Will, Theory and Experiment in Gravitational Physics, Cambridge Univ. Press, Cambridge, 1993.
[25] V. Lukash, Sov. Phys. JETP 40 (1975) 792.
[26] J.D. Barrow, D. Sonoda, Phys. Rep. 139 (1986) 1.
[27] J.D. Barrow, Quart. J. R. Astron. Soc. 23 (1982) 344.
[28] J.D. Barrow, D.H. Sonoda, Gen. Relativ. Gravit. 17 (1985) 409.
[29] J.D. Barrow, H. Kodama, Class. Quantum Grav. 18 (2001) 1753;
J.D. Barrow, H. Kodama, Int. J. Mod. Phys. D (2001).
[30] G.F.R. Ellis, in: R.K. Sachs (Ed.), General Relativity and Cosmology, Academic Press, New York, 1971.
[31] H. Van Elst, Ph.D. Thesis, Extensions and applications of $1+3$ decomposition methods in general relativistic cosmological modelling, available online at: http://www.mth.uct.ac.za/~webpages/henk/abstr.html#THES.
[32] C. Gordon, N. Turok, hep-th/0206138.
[33] J.W. Moffat, hep-th/0208122.
[34] J. Magueijo, L. Pogosian, astro-ph/0211337.
[35] P.C.W. Davies, T.M. Davies, C. Lineweaver, Nature 418 (2002) 602;
M. Duff, hep-th/0208093.